Algebras of analytic functions: from Valencia to Buenos Aires

Joint works with Manolo, Domingo, Santiago Muro and Daniela Vieira

Universidad de Buenos Aires

XIV Encuentros Análisis Funcional Murcia Valencia
Manolo’s birthday
Some definitions:

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We denote by $\mathcal{M}_b(U)$ the spectrum of $H_b(U)$:

$$\mathcal{M}_b(U) = \{ \phi : H_b(U) \to \mathbb{C} : \text{continuous, linear, multiplicative, } \phi \neq 0, \}.$$
The spectrum of $H_b(U)$ - the beginnings


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δ_x(f) = f(x)

If $X$ is symmetrically regular, $M_b(U)$ is a Riemann domain over $X^{**}$.

A particular case: $U = X$ reflexive $M_b(X)$ is a disjoint union of copies of $X$.
$M_b(U) : \text{spectrum of } H_b(U)$
The spectrum of $H_b(U)$ - The beginnings

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$\delta : U \nrightarrow M_b(U)$

$x \mapsto \delta_x$

where $\delta_x(f) = f(x)$

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The spectrum of $H_b(U)$ - the new millenium
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Joint work with Daniel Carando and Domingo García to appear
in Advances in Mathematics.
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$H$ a Hilbert space,

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In almost any (reflexive) Banach space $X$, $\exists U \subset X$ such that:

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For $U = B_{\ell_p}$ the unit ball of $\ell_p$
The spectrum of $H_b(U)$ - next generation

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If $p \in \mathbb{N}$, $p \geq 2$

$M_b(B_{\ell_p})$:
The spectrum of $H_b(U)$ - next generation

For $U = B_{\ell^p}$ the unit ball of $\ell^p$

**If $p \in \mathbb{N}, p \geq 2$**

- For each $0 < \rho < 1$, there are lots of components that are exact copies (via $\pi$) of balls of radius $\rho$ centered at 0.
The spectrum of $H_b(U)$ - next generation

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For $0 < r < 1$, choose $\phi$ a weak-star adherent point of $\{\delta_{r e_n}\}_n$. The component of $\phi$ is a ball of radius $\rho = (1 - r^p)^{1/p}$.
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For $0 < r < 1$, choose $\phi$ a weak-star adherent point of $\{\delta_{r e_n}\}_n$. The component of $\phi$ is a ball of radius $\rho = (1 - r^p)^{1/p}$. What about the other components?
The spectrum of $H_b(U)$ - the South American connection

• This is part of a joint work (in progress) with Santiago Muro and Daniela Vieira (Universidade de São Paulo).
• We started after WidaBA14, a conference held in Buenos Aires.

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$M_b(U)$:
The spectrum of $H_b(U)$ - the South American connection

$X$ a reflexive Banach space with 1-unconditional Schauder basis

- If $U$ is a complete Reinhardt domain...
- ... every component is (a copy of) a complete Reinhardt domain.
The spectrum of $H_b(U)$ - the South American connection

Every component is a ball.

The radius of each component decreases as we move away from $B_{\ell^p}$.

There are two measures of the size of $\phi \in \pi - 1(0)$: $R(\phi)$ and $Q(\phi)$.

The radius $\rho$ of the ball centered at $\phi$ satisfies

$$ (1 - R(\phi))^p \leq \rho \leq (1 - Q(\phi))^p. $$

In particular, the only ball with radius 1 is the ball centered at $\delta_0$. 
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Valencia and Buenos Aires

Valencia

Buenos Aires

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Valencia
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## Valencia and Buenos Aires

<table>
<thead>
<tr>
<th>Valencia</th>
<th>Buenos Aires</th>
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<tbody>
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<td>• Manolo</td>
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- Manolo
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- Pablo G.
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Thank you!

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